

# "BIOSTATISTICS"

24-1-17

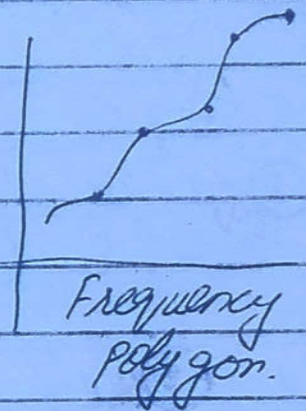
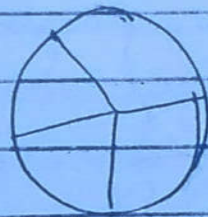
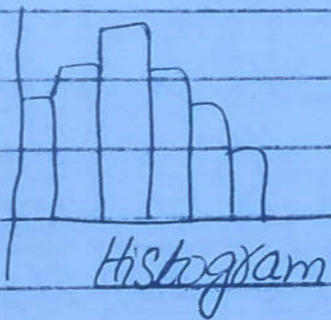
It

Population  $\Rightarrow$  "Total area under discussion"

- Populations by value  $\rightarrow$  ~~Statistics~~ Parameter
- Samples by value  $\rightarrow$  ~~Parameter~~ Statistics

## Qualitative data.

The data which can be measure in the form of quality.



## Measures of dispersion:

Spread of values from

origin

Dispersion of constant value is always '0' zero.

- (i) Absolute dispersion  $\rightarrow$  some unit answer
- (ii) Relative dispersion  $\rightarrow$  unit less answer  
 $\rightarrow$  also known as coefficient of variation.

$\Rightarrow$  Standard deviation is the most important variation of dispersion.

# Lecture #3

31-1-17

$$C.V = \frac{S.d}{\bar{x}} \times 100$$

Range = largest value - smallest value.  
 ↪ Difference b/w two extreme values  
 ↪ low, highest

## Group Data values.

mid values (x)	Classes	f	fx	fx <sup>2</sup> = fxx
smallest value - 2.5	0-5	3	7.5	18.75
7.5	5-10	5	37.5	281.25
12.5	10-15	9	112.5	1406.25
17.5	15-20	8	140	2450
largest 22.5	20-25	2	45	1012.5
		$\Sigma f = 27$	$\Sigma fx = 342.5$	$\Sigma fx^2 = 5168.75$

Range = 22.5 - 2.5

① (a) variance.  $S^2 = \frac{\Sigma fx^2}{\Sigma f} - \left( \frac{\Sigma fx}{\Sigma f} \right)^2$  ✓

② (b)  $S^2 = \frac{\Sigma f(x - \bar{x})^2}{\Sigma f}$

③ Standard Deviation.  $S.D = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left( \frac{\Sigma fx}{\Sigma f} \right)^2}$

④  $C.V = \frac{S.d}{\bar{x}} \times 100$

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{342.5}{27} = 12.685$$

$$\textcircled{1} \quad S^2 = \frac{\sum fx^2}{\sum f} - \left( \frac{\sum fx}{\sum f} \right)^2$$

$$S^2 = \frac{5168.75}{27} - \left( \frac{342.5}{27} \right)^2$$

$$S^2 = 191.43 - (12.68)^2$$

$$S^2 = 191.43 - 160.84$$

$$S^2 = 30.59 = 30.6$$

$$\textcircled{2} \quad S.D. = \sqrt{\frac{\sum fx^2}{\sum f} - \left( \frac{\sum fx}{\sum f} \right)^2}$$

$$S.D. = \sqrt{30.6}$$

$$S.D. = 5.53$$

$$\textcircled{3} \quad C.V. = \frac{S.D.}{\bar{x}} \times 100$$

$$C.V. = \frac{5.53}{12.68} \times 100$$

$$C.V. = 43.61\%$$

$$\text{ii) Mean} = \frac{\sum fx}{\sum f} \quad \boxed{\bar{x}} = \frac{\sum fx}{\sum f}$$

$$\text{iii) Median} = L + \frac{(n/2) - B}{G} \times W$$

$$\text{iii) Mode} = L + \frac{f_m - f_{m-1}}{(f_m - f_{m-1}) + (f_m - f_{m+1})} \times W$$

# Mean deviation (Mean, Median)

mean Deviation from mean

$$\left\{ \begin{array}{l} \text{M.D} = \frac{\sum f |x - \bar{x}|}{\sum f} \quad (\text{Grouped}) \\ \text{M.D} = \frac{\sum |x - \bar{x}|}{n} \quad (\text{Ungrouped}) \end{array} \right.$$

mean deviation from median

$$\left\{ \begin{array}{l} \text{M.D} = \frac{\sum f |x - \tilde{x}|}{\sum f} \quad (\text{Grouped}) \\ \text{M.D} = \frac{\sum |x - \tilde{x}|}{n} \quad (\text{Ungrouped}) \end{array} \right.$$

## Group Data

Classes	f	x	fx	x - $\bar{x}$	f x - $\bar{x}$
0-5	2	2.5	5	12.7	25.4
5-10	3	7.5	22.5	7.7	23.1
10-15	5	12.5	62.5	2.7	13.5
15-20	9	17.5	157.5	2.3	20.7
20-25	4	22.5	90	7.3	29.2
25-30	1	27.5	27.5	12.3	12.3

$$\left. \begin{array}{l} \sum f = 24 \\ \sum fx = 365 \end{array} \right\}$$

$$\left. \begin{array}{l} \sum f|x - \bar{x}| \\ = 124.2 \end{array} \right\}$$

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{365}{24}$$

$$\bar{x} = 15.20$$

$$|x - \bar{x}| = |x - 15.20|$$

these bars change the -ve sign into +ve

$x$	$\bar{x}$	$ x - \bar{x} $
2.5	15.20	12.7
7.5	15.20	7.7
12.5	15.20	2.7
17.5	15.20	2.3
22.5	15.20	7.3
27.5	15.20	12.3

$x$	$\bar{x}$	$ x - \bar{x} $
2.5	31.1	28.6
7.5	31.1	23.6
12.5	31.1	18.6
17.5	31.1	13.6
22.5	31.1	8.6
27.5	31.1	3.6

mean deviation from mean

$$M.D = \frac{\sum f |x - \bar{x}|}{\sum f}$$

$$M.D = \frac{124.2}{24}$$

$$M.D = 5.17$$

$$\bar{x} = 31.1$$

Classes	$x$	O.F	C.F	$x - \bar{x}$	$f x - \bar{x} $
lower class	0-5	2	2	28.6	57.2
	5-10	3	5	23.6	70.8
model class	10-15	5	10	18.6	93
	15-20	9	19	13.6	122.4
	20-25	4	23	8.6	34.4
	25-30	1	24	3.6	3.6
		$\sum f = 24$			

$$\sum f = 24$$

$$\frac{\sum f |x - \bar{x}|}{\sum f} = \frac{384.4}{24} = 16$$

$$\bar{x} = l + \frac{h}{f} \left[ \frac{\sum f x - C}{2} \right]$$

$$\sum f |x - \bar{x}| = 384.4$$

- $C =$  is the previous cumulative frequency of the model class
- $l =$  lower class value of model class
- $f =$  frequency of model class

$h$  = hyphom is the difference b/w two or upper classes boundaries.

$$\bar{x} = l + \frac{h}{f} \left[ \frac{n}{2} - c \right]$$

$$\bar{x} = 15 + \frac{5}{9} \left[ \frac{24}{2} - 10 \right]$$

$$\bar{x} = 15 + 0.55 \left[ 12 - 10 \right]$$

$$\bar{x} = 15 + 0.55 \left[ +2 \right]$$

$$\bar{x} = 31.1$$

Median  
mode

$$M.D = \frac{\sum f |x - \bar{x}|}{\sum f}$$

$$M.D = \frac{381.4}{24}$$

$$M.D = 15.89$$

Modal Class  $\Rightarrow$  The class which have high frequency.

{ Skewness:

{ Kurtosis

Mean; median and mode

Moments:

Pearson:  $S_k = \frac{\text{Mean} - \text{mode}}{S.d}$

i) About mean :-

ii) About any value :-

iii) About zero :-

Bowley's:  $S_k = \frac{Q_3 + Q_4 - 2\bar{x}}{Q_3 - Q_1}$

$Q_1$        $Q_3$   
First and third quartile.

$$Q_1 = l + h \left[ \frac{\frac{n}{4} - c}{f} \right]$$

$$Q_3 = l + h \left[ \frac{\frac{3n}{4} - c}{f} \right]$$

$$\bar{x} = l + h \left[ \frac{\frac{n}{2} - c}{f} \right]$$

$$Q_2 = l + h \left[ \frac{\frac{2n}{4} - c}{f} \right]$$

Quantiles:

- (i) Median
- (ii) Quartiles
- (iii) Deciles
- (iv) Percentiles

Quantiles:

value which divide an array data into some equal parts.

Median:

median is the data value which divide an array data into 2 equal parts.

Quartiles:

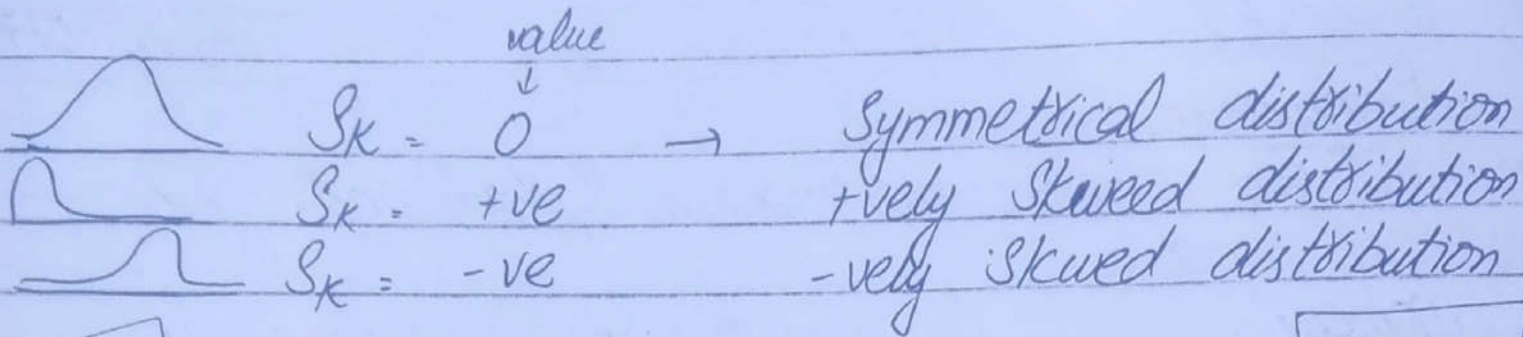
which divide array data into 4 equal parts

Deciles:

which divide - - into 10 equal parts

Percentile:

- - - into 100 equal parts



compares to zero  $\leftarrow \beta_1 = \frac{(m_3)^2}{(m_2)^3}, \beta_2 = \frac{m_4}{(m_2)^2} \rightarrow \begin{matrix} \text{compares} \\ \text{to 3} \end{matrix}$

$m_1, m_2, m_3, m_4 \rightarrow$  are moments of mean.

## Moments About Mean:

$$m_y = \frac{\sum (x - \bar{x})^y}{n}, \quad m_y = \frac{\sum f(x - \bar{x})^y}{\sum f}$$

$y = 1, 2, 3, 4, \dots$

## Moments about any value

$$m'_y = \frac{\sum (x - A)^y}{n}, \quad m'_y = \frac{\sum f(x - A)^y}{\sum f}$$

## moments about zero

$$m'_y = \frac{\sum (x - 0)^y}{n} = \frac{\sum x^y}{n}, \quad m'_y = \frac{\sum f(x - 0)^y}{\sum f} = \frac{\sum fx^y}{\sum f}$$

⇒ First moment about mean is always zero.

$$\bar{X} = \frac{\sum x}{n}$$

$\bar{X} = \frac{34}{5} = 6.8$



Ungroup Data → where no frequency present.

$x$	$(x - \bar{x})$	$(x - \bar{x})^2$	$(x - \bar{x})^3$	$(x - \bar{x})^4$
2	-4.8	<del>(-4.8)² = 23.04</del>	-110.59	530.84
3	-3.8	<del>(-3.8)² = 14.44</del>	-54.87	208.51
5	-1.8	<del>(-1.8)² = 3.24</del>	-5.83	10.49
9	2.2	<del>(2.2)² = 4.84</del>	10.65	23.42
15	8.2	<del>(8.2)² = 67.24</del>	551.368	4521.21
$\Sigma x = 34$	$= 0$	$= 112.8$	$= 390.728$	$= 5294.4$

$$m_n = \frac{\Sigma (x - \bar{x})^n}{n}$$

$$m_1 = \frac{\Sigma (x - \bar{x})^1}{n}$$

$$m_1 = \frac{0}{5} = 0$$

$$m_2 = \frac{\Sigma (x - \bar{x})^2}{n}$$

$$m_2 = \frac{112.8}{5}$$

$$m_2 = \frac{22.56}{5}$$

$$m_4 = \frac{\Sigma (x - \bar{x})^4}{n}$$

$$m_4 = \frac{5294.4}{5}$$

$$m_4 = 1058.84$$

$$m_3 = \frac{\Sigma (x - \bar{x})^3}{n}$$

$$m_3 = \frac{(390.72)}{5}$$

$$m_3 = 78.14$$

$$\beta_1 = \frac{(m_3)^2}{(m_2)^3}$$

$$\beta_1 = \frac{(78.14)^2}{(22.56)^3}$$

$$\beta_1 = \frac{6105.8}{11481.9} = 0.53$$

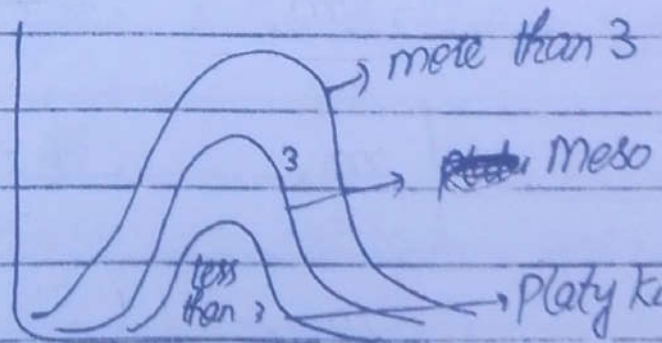
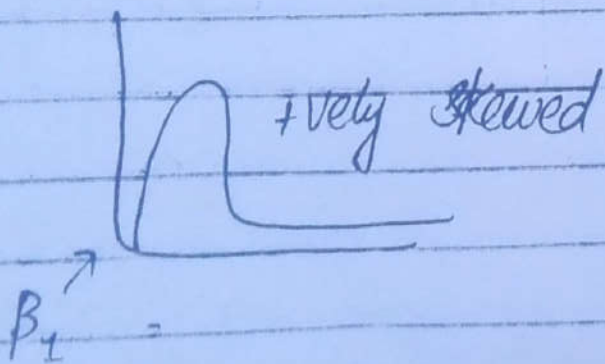
$$\beta_2 = \frac{m_4}{(m_2)^2}$$

$$\beta_2 = \frac{1058.84}{(22.56)^2}$$

$$\beta_2 = 2.08$$

$$\beta_1 = 0.53 \rightarrow \beta_1 \neq 0$$

$$\beta_2 = 2.08 \rightarrow \beta_2 > 3$$



# Lecture # 5

6-2-17

Formulas by which moment about any value and zero are converted into moments about mean.

$$m_1 = m'_1 - m'_1$$

$$m_2 = m'_2 - (m'_1)^2$$

$$m_3 = m'_3 - 3m'_2 m'_1 + 2(m'_1)^3$$

$$m_4 = m'_4 - 4m'_3 m'_1 + 6m'_2 (m'_1)^2 - 3(m'_1)^4$$

Question:

$$m'_1 = 1.3$$

$$m'_2 = 2.6$$

$$m'_3 = 3.1$$

$$m'_4 = 4.2$$

Calculate the  $\beta_1$  and  $\beta_2$ . Also explain results

$$m_1 = m'_1 - m'_1$$

$$m_1 = 1.3 - 1.3$$

$$\boxed{m_1 = 0}$$

$$m_2 = m'_2 - (m'_1)^2$$

$$m_2 = 2.6 - (1.3)^2$$

$$m_2 = 2.6 - 1.69$$

$$\boxed{m_2 = 0.91}$$

$$m_3 = m'_3 - 3m'_2 m'_1 + 2(m'_1)^3$$

$$= 3.1 - 3(2.6)(1.3) + 2(1.3)^3$$

$$= 3.1 - 10.14 + 4.394$$

$$\boxed{m_3 = -2.646}$$

$$\begin{aligned}
 m_4 &= m_4' - 4m_3' m_2' + 6m_2' (m_1')^2 - 3(m_1')^4 \\
 &= 4 \cdot 2 - 4(3 \cdot 1)(1 \cdot 3) + 6(2 \cdot 6)(1 \cdot 3)^2 - 3(1 \cdot 3)^4 \\
 &= 4 \cdot 2 - 16 \cdot 12 + 26 \cdot 364 - 8 \cdot 568 \\
 &= 30 \cdot 364 - 24 \cdot 688
 \end{aligned}$$

$$m_4 = 5.876$$

$$\beta_1 = \frac{(m_3)^2}{(m_1)^3}$$

$$\beta_1 = \frac{(-2.646)^2}{(0.91)^3}$$

$$\beta_1 = \frac{7.00}{0.75}$$

$$\beta_1 = 9.33$$

$$\beta_2 = \frac{m_4}{(m_2)^2}$$

$$\beta_2 = \frac{5.876}{(0.91)^2}$$

$$\beta_2 = \frac{5.876}{0.8281}$$

$$\beta_2 = 7.09$$

Q: Calculate  $\beta_1$ ,  $\beta_2$  and C.V.

Classes	f	x	$(x-\bar{x})$	$(x-\bar{x})^2$	$(x-\bar{x})^3$	$(x-\bar{x})^4$	fx
0-2	2	1	-4.75	22.56	-107.17	509.06	2
2-4	5	3	-2.75	7.56	-20.796	57.19	15
4-6	9	5	0.75	0.56	0.42	0.316	45
6-8	12	7	1.25	1.56	1.953	2.441	84
8-10	3	9	3.25	10.56	34.32	111.56	27
10-12	1	11	5.25	27.56	144.70	759.69	11
	$\Sigma f = 32$	$\Sigma x = 36$					$\Sigma fx = 184$

C.V. =  $\frac{S.d}{\bar{x}} \times 100$       Ans = %       $\Sigma f = 32$

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{184}{32}$$

$$\bar{x} = 5.75$$

$$S.D = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

$$S.D = \sqrt{\frac{1224}{32} - \left(\frac{184}{32}\right)^2}$$

$$S.D = \sqrt{38.25 - (5.75)^2}$$

$$= \sqrt{38.25 - 33.06}$$

$$S.D = \sqrt{5.1875}$$

$$S.D = 2.277$$

$fx^2$	$fx \times x$
$2 \times 1 = 2$	
$15 \times 3 = 45$	
$45 \times 5 = 225$	
$84 \times 7 = 588$	
$27 \times 9 = 243$	
$11 \times 11 = 121$	
$\Sigma fx^2 = 1224$	

$$C.V. = \frac{S.d}{\bar{x}} \times 100$$

$$C.V. = \frac{2.277}{5.75} \times 100$$

$$C.V. = 39.6 \%$$

# Regression and Correlation / Association

**Regression**  $\Rightarrow$  Relation b/w independent and dependent variable.

Defined as dependence of one variable on the independence of one or more variable.

$$\text{dep} \leftarrow \begin{cases} Y = \alpha + \beta X_i + \epsilon_i \\ i = 1, 2, 3, \dots, n \end{cases}$$

Error term  $\leftarrow E(\epsilon_i) = 0$

$$\begin{array}{l} \text{dep} \leftarrow \begin{array}{l} Y = a + bx \\ Y = a_{yx} + b_{yx} X \\ X = a_{xy} + b_{xy} Y \end{array} \end{array}$$

$\xrightarrow{\text{Indep}}$

$a =$  Intercept,  $b =$  Regression coefficient

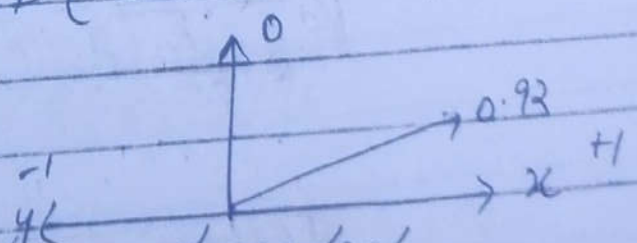
**Correlation**  $\Rightarrow$  Strong relation b/w dependent and independent variable.

Qualitative  $\leftarrow$  attributes.

Correlation = -1 to +1

+ve (direction same), 0 = (No relationship)

-ve (direction opposite)



**Linear**  $\rightarrow$  1 dependent and 1 independent

**Multiple**  $\rightarrow$  Independent 1 and dependent may be many more.

# Properties of Correlation: ~~Properties~~

- ⇒ Correlation always lies b/w -1 to +1
- ⇒ Co-efficient correlation is symmetrical with respect to  $x$  and  $y$ .  $r_{yx} = r_{xy}$
- ⇒ Correlation coefficient is independent from change of origin and scale.  $r_{xy} = r_{uv}$

$$u = \frac{x-A}{h}, \quad v = \frac{y-B}{k} \rightarrow r_{xy} = r_{uv}$$

- ⇒ Correlation coefficient is the geometric mean of two regression coefficients

## Formula of Correlation

$$r_{xy} = r_{yx} \rightarrow r_{xy} = \frac{n \sum xy - (\sum x)(\sum y)}{\sqrt{[n \sum x^2 - (\sum x)^2][n \sum y^2 - (\sum y)^2]}}$$

$$r = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sqrt{\sum (x - \bar{x})^2 \cdot \sum (y - \bar{y})^2}}$$

$$r_{xy} = r_{uv}$$

$$u = \frac{x-A}{h}, \quad v = \frac{y-B}{k}$$

$$r = \frac{\sum (A+hu - A-h\bar{u})(B+kv - B-k\bar{v})}{\sqrt{\sum (A+hu - A-h\bar{u})^2 \cdot \sum (B+kv - B-k\bar{v})^2}}$$

$$x = A+hu, \quad y = B+kv$$

$$\bar{x} = A+h\bar{u}, \quad \bar{y} = B+k\bar{v}$$

$$= \frac{hk \sum (u - \bar{u})(v - \bar{v})}{\sqrt{h^2 k^2 \sum (u - \bar{u})^2 \cdot \sum (v - \bar{v})^2}}$$

$$r_{xy} = \frac{\sum (u - \bar{u})(v - \bar{v})}{\sqrt{\sum (u - \bar{u})^2 \cdot \sum (v - \bar{v})^2}} = r_{uv}$$



x = 6, 11, 9, 13, 5, 16, 8

n = 7

y = 9, 11, 13, 12, 17, 6, 18

$\Sigma y = 86$

$$r_{xy} = \frac{n \Sigma xy - (\Sigma x)(\Sigma y)}{\sqrt{[n \Sigma x^2 - (\Sigma x)^2][n \Sigma y^2 - (\Sigma y)^2]}}$$

x	y	xy	x <sup>2</sup>	y <sup>2</sup>
6	9	54	36	81
11	11	121	121	121
9	13	117	81	169
13	12	156	169	144
5	17	85	25	289
16	6	96	256	36
8	18	144	64	324
$\Sigma x = 68$	$\Sigma y = 86$	$\Sigma xy = 773$	$\Sigma x^2 = 752$	$\Sigma y^2 = 1164$

$$r_{xy} = \frac{7(773) - (68)(86)}{\sqrt{[7(752) - (68)^2][7(1164) - (86)^2]}}$$

$$r_{xy} = \frac{5411 - 5848}{\sqrt{[5264 - 4624][8148 - 7396]}}$$

$$r_{xy} = \frac{-437}{\sqrt{(640)(752)}} = \frac{-437}{\sqrt{481280}}$$

$$r_{xy} = \frac{-437}{722.71} = -0.62$$